DAWSON COLLEGE
Mathematics Department

FINAL EXAMINATION

Probability and Statistics (201-BZS-05)

Fall 2021

Instructor:
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Student Name: _____

Student ID. #: _____

Instructions:

- You are required to show your work on each problem, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering could receive very little credit.
- Round your answers to the at least four decimal places.
- For the Test of Hypothesis questions, you need to write the exact H_0 and H_α, you need also clearly write the final result and decision.
- You are only allowed to use a calculator Sharp EL 531.X/ XG/XT.
- The formula sheet and the tables are at the end of the examination booklet and must be returned intact.

This examination consists of 18 questions. Please ensure that you have a complete examination booklet before starting.

Q1-Q8/16	Q9/8	Q10/9	Q11/10	Q12/10	Q13/6	Q14/10	Q15/6	Q16/8	Q17/8	Q18/9	Sum/100

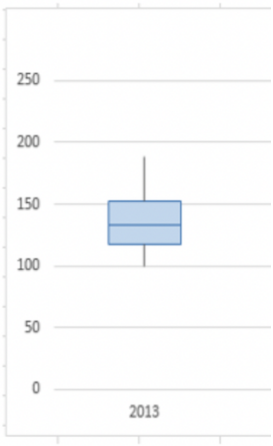
(Each Question 2 marks) For the questions 1-8, write only the final answers in the given boxes. For these problems, the marks are only considered for the right final answers.

(1) Find the level of measurement of the following variables.

IQ (Intelligence Quotient)

Time to respond to a question

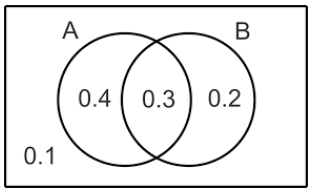
(2) The given box-plot is constructed based on the number of weekly advertisements for a company in 2013.
Find the probability of those weeks for which the number of ads were less than 150.



(3) If a single die is rolled twice, what is the probability of getting the sum of two outcomes less than 10?

(4) A card is randomly selected from a deck of 52 cards. Find the probability that it is a red face or a heart? (Face cards consist of Kings, Queens and Jacks.)

(5) Find the conditional probability $P(A|B)$ from the following Venn diagram. The numbers indicate the probability of different parts.



(6) For the same Venn diagram given in the previous question, Find $P(A|B^c)$

(7) Consider the normal probability distribution $X \sim \mathcal{N}(1200, 125)$. Find $P(1320 < X)$.

(8) Consider the normal probability distribution $X \sim \mathcal{N}(1200, 125)$. Find $P(1050 < X \leq 1320)$.

For the following questions (Questions 9-18) show all your work with the details.

(9) You have been in the walking/jogging exercise program for 26 weeks, and for each week you have recorded the distance (in miles) you covered in 30 minutes.

1.5	1.5	1.4	1.7	1.6	1.9	1.9	2.0	1.8	2.0	1.9	2.0	2.0
1.9	2.1	2.1	2.3	2.3	2.2	2.4	2.5	2.6	2.4	2.6	2.5	2.7

(a) (6 marks) Construct a frequency distribution table using 5 classes. **Make sure to consider the accuracy of the data**.

class limits	class boundaries	class marks	frequency	cumulative frequency

(b) (2 marks) Find P_{35} and Q_2.

(10) Tim's Hardware store manager believes that there is a linear relationship between the income from sales of goods (Y, in thousands of dollars) and the amount spent on advertising (X, in thousands of dollars).

Advertising Spending	Income from Sales
5.3	21
3.8	16
3.1	13
2.9	12
4.4	23
4.9	20
5.1	23
5.4	24
3.2	14
5.1	19

The table gives us :

$\sum x = 43.2, \quad \sum x^2 = 195.34$

$\sum y = 185 \quad \sum y^2 = 3601 \quad \sum xy = 835$

(a) (5 marks) Find the least squares regression line.

(b) (2 marks) Use the answer of part (a) to Predict that how much do they need to spend on advertising if they want to have $25000 income sales.

(c) (2 marks) Calculate the correlation coefficient and comment on the accuracy of the estimate obtained in part (b).

(11) The Notre Dame Model UN club has 20 members. Five are seniors, four are juniors, two are sophomores and nine are freshmen.

(a) (2 marks) In how many ways can the club select a president, a secretary and a treasurer if every member is eligible for each position and no member can hold more than one position?

(b) (2 marks) In how many ways can the club choose a group of 5 members to attend the next Model UN meeting in Washington.

(c) (3 marks) In how many ways can the club choose a group of 5 members to attend the next Model UN meeting in Washington if at least two of them must be freshmen.

(d) (3 marks) In how many ways can the club choose a group of 5 members to attend the next Model UN meeting in Washington if at least one member from each class (Seniors, Juniors, Sophomores, Freshmen).

(12) A lucky dip at a school fete contains 60 packages of which 10 contain tickets for prizes. Let X denote the number of prizes you win when you draw out **three** of the packages at once (without replacement).

(a) (8 marks) Find the probability density of X i.e.$P(X = i)$ for each appropriate i.

(b) (2 marks) Find $E(X)$.

(13) **(6 marks)** In a major city 8% of cars have an alarm system installed. A car without an alarm has a 0.03 probability of being stolen, whereas a car with an alarm has the probability of being stolen is 0.01. If a car from this major city is stolen what is the probability that it had an alarm system installed?

(14) In the United States, 0.6% of the population is allergic to peanuts. A random sample of 2000 people will be selected. Let X be the number of people in our sample who has allergy to peanuts.

(a) (2 marks) What is the appropriate distribution for X? (Write the distribution and the parameters associate to that)

(b) (4 marks) Find $E(X)$ and $Var(X)$.

(c) (4 marks) Use normal distribution to approximate $P(10 < X < 50)$.

(15) Births in a hospital occur randomly at an average rate of 1.8 births per hour.

(a) (3 marks) What is the probability that no birth happens tomorrow morning from 8- a.m. to 10 a.m.?

(b) (3 marks) What is the probability to have more than 2 births tomorrow morning from 8- a.m. to 10 a.m.?

(16) Suppose that out of 14,750 convicts who escaped from North American prisons, only 7055 were recaptured. Let p represent the proportion of all escaped convicts who will eventually be recaptured.

(a) (3 marks) Find a 99% confidence interval for p.

(b) (5 marks) Use p-value to test the claim that the true proportion of convicts that are recaptured is less than 50%.

(17) In a packing plant, a machine packs cartons with jars. It is supposed that a new machine packs faster on the average than the machine currently is used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results are, in seconds and are shown in the table below.

Machine	Sample Size	Sample Mean	Sample Variance
New	10	42.24	0.653
Old	10	43.33	0.745

(a)(7 marks) Do the data provide sufficient evidence to conclude that, on the average, the new machine packs faster? Perform the required hypothesis test at the 2.5% level of significance using the rejection region approach. Assume both samples come from normal distributions with the same variances.

(b) (1 mark) Find $P(H_0 \textit{ is rejected} \mid H_0 \textit{ is true.})$.

(18) **(9 marks)** There are three sections of "Probability and Statistics" offered by three different teachers last year in a college. Determine whether the proportions of passed and failed students are independent of their teachers, if their results are

	Teacher A	Teacher B	Teacher C
Passed	31	28	33
Failed	4	8	6

Conduct an independent classification test using 0.05 level of significance.

Answers

(1) Interval; Ratio

(2) 75%

(3) $\frac{5}{6}$

(4) $\frac{4}{13}$

(5) $\frac{3}{5}$

(6) $\frac{4}{5}$

(7) 0.1685

(8) 0.7164

(9) **(a)**

class limits	class boundaries	class marks	frequency	cumulative frequency
1.4-1.6	1.35-1.65	1.5	4	4
1.7-1.9	1.65-1.95	1.8	6	10
2.0-2.2	1.95-2.25	2.1	7	17
2.3-2.5	2.25-2.55	2.4	6	23
2.6-2.8	2.55-2.85	2.7	3	26

(b) $P_{35} = 1.9,\ Q_2 = 2.0$

(10) **(a)** $y = 0.75608 + 4.10739x$

(b) 5.9025 thousand dollars

(c) $r^2 = 82.378\%$

(11) **(a)** 6840

(b) 15504

(c) 12072

(d) 2880

(12) **(a)**

X	P(X=x)
0	0.57276
1	0.35798
2	0.065751
3	0.003507

(b) $E(X) = 0.5000$

(13) 0.028169

(14) **(a)** $\binom{n}{x} 0.006^x \ 0.994^{(n-x)}$

(b) $E(X) = 12, \ Var(X) = 11.928$

(c) 0.6664

(15) **(a)** 0.0273237

(b) 0.697254

(16) **(a)** $[0.46771, 0.48889]$

(b) p-value=-5.27 $\longrightarrow$ We can support the claim.

(17) **(a)** $t = -2.92 \ \longrightarrow$ At this level of significance, the new machines pack faster than the old ones (supporting the claim).

(b) P(type I error)=0.025

(18) $\chi_0^2 = 1.5295 \ \longrightarrow$ Fail to reject H_0.